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Обучение чтению литературы на английском языке по специальности «Прикладная математика»

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Учебное пособие состоит из трех уроков и содержит современные неадаптированные тексты, заимствованные из оригинальных источников, о сущности прикладной математики как науки и ее связях с другими науками. Каждый из уроков включает базовый текст А, упражнения на контроль понимания текстов, подбор активной лексики, перевод с русского на английский язык, умение читать математические формулы, а также дополнительные тексты для различных видов работ с ними. Грамматические упражнения стимулируют повторение наиболее сложных грамматических конструкций. В Приложении приведен перечень основных математических символов и формул, а также даны варианты правильного их прочтения на английском языке.

Для студентов старших курсов факультета «Фундаментальные науки», обучающихся по специальности «Прикладная математика».

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ПРЕДИСЛОВИЕ

Целью учебного пособия является развитие у студентов старших курсов, обучающихся по специальности «Прикладная математика», навыков работы с оригинальной научной литературой на английском языке, а также точного понимания и грамотного перевода текстов, ведения беседы по основным темам, затронутым в пособии. Задания по переводу с русского языка на английский направлены на повторение и закрепление терминологии по специальности, на использование необходимого грамматического аппарата. Задания на прочтение математических формул позволят студентам получить достаточный навык изложения материала, содержащего математический аппарат, на английском языке. Грамматические упражнения направлены на повторение наиболее сложных конструкций английского языка.

Владение терминологией по изучаемой специальности и языковыми оборотами английского языка, навыки понимания и перевода оригинальной литературы позволят студентам легче ориентироваться в потоке публикаций по специальности на английском языке, определять степень важности получаемой информации для собственной сферы деятельности, принимать участие в обсуждении профессиональных вопросов с зарубежными коллегами.

Unit 1

Texts:

A. Pure and Applied Mathematics
B. Why is the World Mathematical?
C. Mathematics and Physics
D. Revolution in Mathematics
E. "Queen of Sciences"
F. Experimental Mathematics

Preliminary exercises

I. Translate the following words and determine what part of speech they are. Explain your opinion. Find them in the text:
National, international, nation, nationality, nationalism, nationalist, nationally;
discuss, discusser, discussible, discussion;
approximately, approximation, approximate;
solve, solvent, solvable, solver;
application, applicator, applicant, apply, appliancy, applicability, applied, appliance;
extremely, extreme, extremeness, extremism, extremist, extremity;
intractable, tract, tractable, tractate;
essential, essence, essentially, essay;
heavily, heavy, heaviness;
mathematicians, mathematics, mathematical, math;
probabilists, probable, probability;
correctly, correct, correction, incorrect, incorrectly, correctness;
joining, join, joint, jointless, jointly, joined;
partition, part, particle, partial, partner, partly;
loft, loftiness, loftily, lofty.

II. Read Text A and find equivalent phrases in the right-hand column. Find them in the text:

1) строгость и доказательства		a) solvable problem

2) начальные и граничные значения	b) sloppy crackpot
3) теоретическая и прикладная математика отдаляются друг от друга	c) to immerse oneself completely in the subject
4) глубокое понимание предмета	d) joining of hands of people
5) индуктивный метод	e) initial and boundary values
6) разрешимая задача	f) living and breathing the subject
7) полностью погрузиться в тему	g) pure and applied mathematics are drifting apart
8) необходимые качества	h) inductive method
9) объединенные усилия людей	i) rigor and proofs
10) неграмотный чудак	j) requisite qualities

III. Memorize the following basic vocabulary and terminology to Text A:

pure mathematics — чистая математика
applied mathematics — прикладная математика
boundary value — граничное значение
boundary value problem — краевая задача
approximate solution — приближённое решение
exact problem — строгая задача
solvable problem — разрешимая задача
intractable problem — трудноразрешимая задача
initial value problem — задача Коши, задача с начальными условиями
recognize the need — признавать необходимость

Text A

Pure and Applied Mathematics

Toward the end of the recent International Congress of Mathematicians in Madrid, there was a discussion about whether pure and applied mathematics are drifting apart. The majority of the audience was pure mathematicians. So perhaps it would be helpful to ask, what is applied mathematics?

A very good answer was provided by Kurt Friedrichs, who distinguished himself in both pure and applied mathematics, “Applied mathematics consists in solving exact problems approximately and approximate problems exactly." Initial and boundary value problems associated with the Navier-Stokes equations are an example of problems that are extremely difficult to solve exactly and where approximate solutions are looked for. Hence computing is an important part of applied mathematics. The Bhatnagar Gross-Krook equation in kinetic theory and plasma physics is an example of a solvable problem that approximates an intractable one.

Some mathematicians believe that pure mathematics is a branch of applied mathematics. Some of the greatest mathematicians of the past — Newton, Euler, Lagrange, Gauss, and Riemann — and more recently Hilbert, Weyl, Wiener, von Neumann, and Kolmogorov did both pure and applied mathematics.

There was the opinion that proofs are essential in pure math, but they are essential in applied math too, except that the path one takes is rather different. Applied math relies heavily on the inductive method, as opposed to the deductive method preferred by pure mathematicians. In pure mathematics the emphasis is on rigor. However, ideas are far more important. Ideas come from intuition, of course, which in turn comes from living and breathing the subject. Some of the scientists are quite right in insisting that even applied mathematicians need basic training in mathematics. One must also immerse oneself completely in the subject to which one wants to apply mathematics.

It is by gaining a thorough understanding of the problems arising in the subject one develops a feeling for it, and with it, intuition. In applied mathematics the emphasis on rigor and proof must come at the appropriate stage. Let us consider an example. Feynman had great intuition but didn't care much for rigor or proofs. He says in one of his autobiographical writings that once he used to talk to William Feller and Mark Kac, the famous probabilists. It is a happy circumstance, for science in general and mathematics in particular, that Feller and Kac didn't dismiss Feynman as a sloppy crackpot but instead patiently listened to him. Thus the great Feynman-Kac formula was born. The moral, I think, is that pure mathematicians, while insisting correctly on rigor and proofs, must be patient and show some respect toward intuition born out of a deep knowledge of a subject. Attitudes like "Applied mathematics is bad mathematics" are shortsighted. For their part, applied mathematicians, while using intuition as their guide, must recognize the need for and the importance of proofs. On the other hand, it is rare that a single individual embodies all the requisite qualities to a high degree. So often what is needed is a joining of hands of people with disparate abilities, strengths, and points of view rather than a separation or drifting apart.

(*3 253*)

Task 1. ***Answer the following questions.***

1. What was one of the questions discussed at the recent International Congress of Mathematicians in Madrid?
2. How did Kurt Friedrichs define the difference between pure and applied mathematics? What example could prove his point?
3. What does applied mathematics require besides basic training in mathematics?
4. Thanks to what circumstances was the famous Fayman-Kac formula born? What is proved by that fact?
5. What is the source of intuition?
6. Why is the joining of hands of all mathematicians necessary in dealing with problems of mathematics?

Task 2. ***Translate the following sentences into English.***

1. Прикладная математика состоит в приближенном решении точных задач и точном решении приближенных. 2. Существует мнение, что доказательства очень важны в теоретической математике. 3. Только в результате глубокого понимания проблем, возникающих при рассмотрении конкретной темы, у человека развивается его тонкое восприятие, и с ним, интуиция. 4. Те, кто, занимаясь прикладной математикой, руководствуется интуицией, должны в то же время осознавать необходимость доказательств и их важность.

Task 3. ***Read the following expressions, consulting the SUPPLEMENT.***

$1\frac{3}{8}$; 0.12; 9.43.

$2x^2 + 7x = 0$; $10^{a+5} - 3 = 2$; $2x + 3 = \sqrt{20x+9}$; $\sqrt[3]{2+11x^2} = 3 + x$;

$\mathrm{K} = \frac{1}{2}\sqrt{\frac{N_{T+R_2}}{R_1}}$, $\frac{a^4 - b^4}{a^2 - b^2} = a^2 + b^2$.

$log \sqrt[m]{N} = \frac{logN}{m}$; $2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$;

$\int \frac{dx}{3-5x}$; $\int_{-a}^{a} \sqrt{a^2 - x^2}\, dx$; $\int_{\frac{\pi}{2}}^{\pi} \frac{\sin x\, dx}{\cos^2 x + 1}$.

Grammar Revision

Grammar task 1. ***Find the passive constructions in Text A and explain them.***

Grammar task 2. ***Translate the following sentences into Russian paying attention to the passive constructions.***

1. The question of the laws of resistance in circuits may now be turned to.
2. Many materials now commonly used were not even thought of forty years ago.
3. This result was aimed at.
4. Mathematics, astronomy and physics were the first sciences to get organized and defined.
5. The speed with which arithmetic operations are performed is affected by a number of factors.
6. Questions can be asked and answered, but unfortunately the questions asked and those answered are frequently not the same.
7. These problems were being discussed by physicists for many years.
8. The equipment was sent for.
9. The force was acted upon.
10. Advantage was taken of this fact.
11. Use is being made of the new technique developed by the young engineer.
12. Care should be taken of the exact following the instructions.
13. This question was very important but not paid due attention to.
14. The weak points in the thesis were not taken notice of.
15. The young man left the city and was lost sight of.
16. Materials can be classed in three groups according to their electrical properties — conductors, semiconductors and insulators.
17. The results of the Dubna physicist research work are made good use of in such fields as biology, medicine, geology and science of metals.

18. An atom of any substance may be represented by a central core having a positive charge and surrounded by orbiting electrons, each having a negative charge.
19. Granules cannot be obtained from such metals.
20. The book was terribly bad; it was just a chance that it got published.

Supplementary reading tasks

Read and translate Text B without a dictionary

Text B

Why is the World Mathematical?

This reflection on the symmetries behind the laws of nature also tells us why mathematics is so useful in practice. Mathematics is simply the catalogue of all possible patterns. Some of those patterns are especially attractive and are studied or used for decorative purposes; others are patterns in time or in chains of logic. Some are described solely in abstract terms, while others can be drawn on paper or carved in stone. Viewed in this way, it is inevitable that the world is described by mathematics. We could not exist in a universe in which there was neither pattern nor order. The description of that order (and all the other sorts that we can imagine) is what we call mathematics. Yet, although the fact that mathematics describes the world is not a mystery, the exceptional utility of mathematics is. It could have been that the patterns behind the world were of such complexity that no simple algorithms could approximate them. Such a universe would "be" mathematical, but we would not find mathematics terribly useful. We could prove "existence" theorems about what structures exist, but we would be unable to predict the future using mathematics in the way that NASA's mission control does.

Seen in this light, we recognize that the great mystery about mathematics and the world is that such simple mathematics is so far-reaching. Very simple patterns, described by mathematics that is easily within our grasp, allow us to explain and understand a huge part of the universe and the happenings within it.

(*1520*)

Read Text C and give a short summary.

Text C

Mathematics and Physics

The traditional view is that mathematics and physics are quite different. Physics describes the universe and depends on experiment and observation. The particular laws that govern our universe — whether Newton's laws of motion or the Standard Model of particle physics — must be determined empirically and then asserted like axioms that cannot be logically proved, merely verified. Mathematics, in contrast, is somehow independent of the universe. Results and theorems, such as the properties of the integers and real numbers, do not depend in any way on the particular nature of reality in which we find ourselves. Mathematical truths would be true in any universe. Yet both fields are similar. In physics and indeed in science generally, scientists compress their experimental observations into scientific laws. They then show how their observations can be deduced from these laws. In mathematics, too, something like this happens — mathematicians compress their computational experiments into mathematical axioms, and they then show how to deduce theorems from these axioms. If Hilbert had been right, mathematics would be a closed system, without room for new ideas. There would be a static, closed theory of everything for all of mathematics, and this would be like a dictatorship. In fact, for mathematics to progress you actually need new ideas and plenty of room for creativity. It does not suffice to grind away, mechanically deducing all the possible consequences of a fixed number of basic principles. An open system is much more preferable. Rigid, authoritarian ways of thinking are ineffective. Another person who thought mathematics is like physics was Imre Lakatos, who left Hungary in 1956 and later worked on philosophy of science in England. There Lakatos came up with a great word, "quasiempirical," which means that even though there are no true experiments that can be carried out in mathematics, something similar does take place. For example, the Goldbach conjecture states that any even number greater than 2 can be expressed as the sum of two prime numbers. This conjecture was arrived at experimentally, by noting empirically that it was true for every even number that anyone cared to examine. The conjecture has not yet been proved, but it has been verified up to 1014. It appears that mathematics is quasiempirical. In other words, it seems that mathematics is different from physics (which is truly empirical) but perhaps not as different as most people think. It is a matter of degree, of emphasis, not an absolute difference. After all, mathematics and physics coevolved. Mathematicians should not isolate themselves. They should not cut themselves off from rich sources of new ideas.

(*2712*)

Read Text D and put your own questions to the text. Discuss the questions with the group.

Text D

Revolution in Mathematics

Physical sciences all went through “revolutions”: wrenching transitions in which methods changed radically and became much more powerful. It is not widely realized, but there was a similar transition in mathematics between 1890 and 1930.

To a first approximation the method of science is “find an explanation and test it thoroughly”, while modern core mathematics is “find an explanation without rule violations”. The criteria for validity are radically different: science depends on comparison with external reality, while mathematics is internal.

The conventional wisdom is that mathematics has always depended on error-free logical argument, but this is not completely true. It is quite easy to make mistakes with infinitesimals, infinite series, continuity, differentiability, and so forth, and even possible to get erroneous conclusions about triangles in Euclidean geometry. When intuitive formulations are used, there are no reliable rule-based ways to see these are wrong, so in practice ambiguity and mistakes used to be resolved with external criteria, including testing against accepted conclusions, feedback from authorities, and comparison with physical reality. In other words, before the transition mathematics was to some degree scientific.

The breakthrough was the development of a system of rules and procedures that really worked, in the sense that, if they are followed very carefully, then arguments without rule violations give completely reliable conclusions. It became possible, for instance, to see that some intuitively outrageous things are nonetheless true. Weierstrass’s no-where-differentiable functions and Peano's horrifying space-filling curve were early examples, and we have seen much stranger things since. There is no abstract reason that such a useful system of rules exists, and no assurance that we would find it. However, it does exist and, after thousands of years of tinkering and under the pressure from sciences for substantial progress, we did find it. Major components of the new methods are:

Precise definitions: Old definitions usually described what things are supposed to be and what they mean, and extraction of properties relied to some degree on intuition and physical experience. Modern definitions are completely self-contained, and the only properties that can be ascribed to an object are those that can be rigorously deduced from the definition.

Logically complete proofs: Old proofs could include appeals to physical intuition, authority (e.g. "Euler did this so it must be OK"), and casual establishment of alternatives. Modern proofs require each step to be carefully justified.

It took while to learn to use modern definitions, to see how to pack wisdom and experience into a list of axioms, how to fine-tune them to optimize their properties and how to see opportunities where a new definition might organize a body of material.

Also it took while to learn using logically complete proofs. The "official" description as a sequence of statements obtained by logical operations, and so forth, is cumbersome and opaque, but ways were developed to compress and streamline proofs without losing reliability.

(*3150*)

Read Text E, translate it into Russian and render.

Text E

"Queen of Sciences"

Mathematics was coined the "queen of sciences" by the "prince of mathematicians," Carl Friedrich Gauss, one of the greatest mathematicians of all time. Indeed, the name of Gauss is associated with essentially all areas of mathematics, and in this respect, there is really no clear boundary between "pure mathematics" and "applied mathematics." To ensure financial independence, Gauss decided on a stable career in astronomy, which is one of the oldest sciences and was perhaps the most popular one during the eighteenth and nineteenth centuries. In his study of celestial motion and orbits and a diversity of disciplines later in his career, including (in chronicle order): geodesy, magnetism, dioptrics, and actuarial science, Gauss has developed a vast volume of mathematical methods and tools that are still instrumental to our current study of applied mathematics.

During the twentieth century, with the exciting development of quantum field theory, with the prosperity of the aviation industry, and with the bullish activity in financial market trading, and so forth, the sovereignty of the "queen of sciences" has turned her attention to the theoretical development and numerical solutions of partial differential equations (PDEs). Indeed, the non-relativistic modeling of quantum mechanics is described by the Schrödinger equation; the fluid flow formulation, as an extension of Newtonian physics by incorporating motion and stress, is modeled by the Navier-Stokes equation; and option stock trading with minimum risk can be modeled by the Black-Scholes equation. All of these equations are PDEs. In general, PDEs are used to describe a wide variety of phenomena, including: heat diffusion, sound wave propagation, electromagnetic wave radiation, vibration, electrostatics, electrodynamics, fluid flow, and elasticity, just to name a few. For this reason, the theoretical and numerical development of PDEs has been considered the core of applied mathematics, at least in the academic environment.

(*2002*)

Read Text F and translate it into Russian.

Text F

Experimental Mathematics

Another area of similarity between mathematics and physics is experimental mathematics: the discovery of new mathematical results by looking at many examples using a computer. Whereas this approach is not as persuasive as a short proof, it can be more convincing than a long and extremely complicated proof, and for some purposes it is quite sufficient.

In the past, this approach was defended with great vigor by both George Pólya and Lakatos, believers in heuristic reasoning and in the quasi-empirical nature of mathematics. This methodology is also practiced and justified in Stephen Wolfram's "A New Kind of Science" (2002).

Extensive computer calculations can be extremely persuasive, but do they render proof unnecessary? Yes and no. In fact, they provide a different kind of evidence. In important situations, it seems doubtful that both kinds of evidence are required, as proofs may be flawed, and conversely computer searches may have the bad luck to stop just before encountering a counterexample that disproves the conjectured result. All these issues are intriguing but far from resolved. Now it is not clear how serious incompleteness is. We do not know if incompleteness is telling us that mathematics should be done somewhat differently. Maybe 50 years from now we will know the answer.

(*1300*)

Read and smile

Mathematics is made of 50 percent formulas, 50 percent proofs, and 50 percent imagination.

"A mathematician is a device for turning coffee into theorems" (P. Erdos) Addendum: American coffee is good for lemmas.

Unit 2

Texts:

A. What is the World Like?
B. Outcomes of the Laws of Nature
C. Complexity and Scientific Laws
D. Disorganized Complexities
E. On the Edge of Chaos
F. The Number Omega

Preliminary exercises

I. Read Text A and choose the relevant meaning of the following words:

alternative, *n* — выбор, вариант, возможность;
alternative, *adj*. — другой, взаимоисключающий, переменный, знакопеременный, чередующийся

claim, *n* — утверждение, заявка, заявление, требование;
claim, *v* — утверждать, заявлять, требовать

spring, *n* — пружина, прыжок, скачок, разбег, упругость, родник, источник;
spring, *v* — отпускать пружину, прыгать, появляться, брызгать, наброситься

regularity, *n* — закономерность, правильность, регулярность, равномерность

pattern, *n* — структура, образец, система, пример, схема, диаграмма, типовой вариант, характер, картина, закономерность;
pattern, *v* — структурировать, делать по образцу, располагать по схеме, формировать (картину, изображение), служить образцом

update, *n* — уточнение, уточненный вариант, обновление, новейшая информация, последний вариант, свежие новости;
update, *v* — уточнять, вносить изменения, дополнять, наращивать, обновлять, доводить до современного уровня

recurrence, *n* — возврат, рекурсия, возвращение, повторение, многократность, периодичность, цикличность;

recurrence, *adj*. — рекуррентный, повторяющийся

uniquely, *adv*. — однозначно, уникально, единственно, единственным образом

invariance, *n* — неизменность, неизменяемость, инвариантность, сохранение, сохранность, постоянство

II. Find equivalent phrases in the right-hand column. Find them in the text:

1) законы одинаковы везде	a) a mess of complex events
2) комплекс математических инструментов	b) in the long run
3) искать закономерность в событиях	c) the most significant advance
4) определение закона тяготения	d) the laws were the same everywhere
5) возможные закономерности	e) a battery of mathematical tools
6) нагромождение сложных событий	f) to look for patterns in the facts
7) закон причины и следствия	g) identification of the law of gravitation
8) самый значительный прогресс	h) possible patterns
9) в долгосрочной перспективе.	i) law of cause and effect

III. Memorize the following basic vocabulary and terminology to Text A:

simplicity — простота
observation — наблюдение
regularity — закономерность
law of gravitation — закон всемирного тяготения
to document — документировать, фиксировать, подтвердить документами
content — суть, основное содержание, объём, вместимость, ёмкость
to be content — довольствоваться, удовлетворяться
to predict — прогнозировать
likelihood — вероятность, правдоподобие
recurrence — многократное повторение, повторяемость, рекуррентность
invariance — инвариантность, неизменность, сохранность

Text A

What is the World Like?

Is the world simple or complicated? As with many things, it depends on whom you ask, when you ask, and how seriously they take you. If you should ask a particle physicist, you would soon be hearing how wonderfully simple the universe appears to be. For the psychologist, the economist, or the botanist, the world is a mess of complex events that just seemed to win out over other alternatives in the long run. So who is right? Is the world really simple, as the particle physicists claim, or is it as complex as almost everyone else seems to think? Understanding the question, why you got two different answers, and what the difference is telling us about the world, is a key part of the story of science over the past 350 years.

Our belief in the simplicity of nature springs from the observation that there are regularities which we call "laws" of nature. The idea of laws of nature has a long history. The most significant advance in our understanding of the nature and consequences followed Newton's identification of a law of gravitation in the late seventeenth century, and his creation of a battery of mathematical tools with which to determine its consequences.

Laws reflect the existence of patterns in nature. We might even define science as the search for those patterns. We observe and document the world in all possible ways; but while this data-gathering is necessary for science, it is not sufficient. We are not simply to acquire a record of everything that has ever happened, like cosmic stamp collectors. Instead, we look for patterns in the facts, and some of these patterns we have come to call the laws of nature, while others have achieved only the status of by-laws. Having found (or guessed — for there are no rules at all about how you might find them) possible patterns, we use them to predict what should happen if the pattern is also followed at all times and in places where we have yet to look. Then we check if we are right (there are strict rules about how you do this!). In this way, we can update our candidate pattern and improve the likelihood that it explains what we see. Sometimes likelihood gets so low that we say the proposal is "falsified," or so high that it is "confirmed" or "verified," although strictly speaking neither is ever possible with certainty. This is called the “scientific method”. For Newton and his contemporaries the laws of motion were codifications into simple mathematical form of the habits and recurrences of nature. They were idealistic: “bodies acted upon by no forces will …,” because there are no such bodies. They were laws of cause and effect: they told you what happened if a force was applied. The present state determined the future uniquely and completely.

Later, these laws of change were found to be equivalent to statements that some given quantity was unchanging: the requirement that the laws were the same everywhere in the Universe was equivalent to the conservation of momentum; the requirement that they were found to be the same at all times was equivalent to the conservation of energy; and the requirement that they were found the same in every direction in the Universe was equivalent to the conservation of angular momentum. This way of looking at the world in terms of conserved quantities, or invariances and unchanging patterns, would prove to be extremely fruitful.

(*3 371*)

Task 1. ***Answer the following questions.***

1. If you ask a particle physicist whether the world is simple or complex, what answer will he give? Why do you think?
2. Who will not agree with the particle physicist?
3. What does our belief in the simplicity of nature come from?
4. What was the most significant advance in our understanding of nature?
5. Data gathering may be considered the first step in finding a law of nature. Is it sufficient for science? What must be done next?
6. Why could Newton’s laws of motion be called “idealistic”?

Task 2. ***Translate the following sentences into English.***

1. Если спросить физика-ядерщика, психолога, экономиста или ботаника, является ли мир простым или сложным, ответы будут совершенно разными. 2. Ответ зависит от того, кого вы спрашиваете, когда вы спрашиваете, и насколько серьезно они к вам относятся. 3. Нас не удовлетворяет просто получение перечня когда-либо произошедших событий. 4. Мы ищем закономерности в событиях, и некоторые из этих закономерностей мы привыкли называть законами. 5. Законы отражают существование закономерностей в природе.

Task 3. ***Read the following expressions, consulting the SUPPLEMENT.***

$4\frac{5}{6}$; 0.63; 7.21

$3x^2 + 7x = 15$; $3^{2a} - a \cdot 5^a + 6 = 0$; $\sqrt[4]{3a+7} - 1 = \sqrt{6x-17}$;

$b = \sqrt{\frac{(a_1 - a)^2 + (a_2 - a)^2}{2}}$; $\frac{a^3 + b^3}{a+b} = a^2 - \mathrm{ab} + b^2$;

$\ln N = \frac{\lg N}{\lg e}$; $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$;

$\int 2^{3x-1}dx$; $\int_0^1 \frac{x}{(3x^2-1)^4}dx$; $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}dx$.

Grammar Revision

Grammar task 1. ***Find the infinitive construction in Text A and explain them.***

Grammar task 2. ***Translate the following sentences into Russian paying attention to the infinitive constructions.***

1. He appeared to have plenty of money, which was said to be gained in the California goldfields.
2. Every feature seemed to have sharpened since he saw her last.
3. The house appeared to have been repaired recently.
4. I suppose Mr. Jelleby had been more talkative and lively once; but he seemed to have exhausted long before I knew him.
5. For the last few days she seemed to talk to nobody but strange men.
6. I lack the will-power to do anything with my life, to better my position by hard work.
7. It was the first time he had ever seen her weep.
8. I came to get someone to tell me the truth.
9. He looked at his watch, rang the bell, and ordered the vehicle to be brought round immediately.
10. Young men of this class never do anything for themselves that they can get other people do for them.

Read Text B and put your own questions to the text. Discuss the questions with the group.

Text B

Outcomes of the Laws of Nature

The simplicity and economy of the laws and symmetries that govern nature's fundamental forces is not the end of the story. When we look around us we do not observe the laws of nature; rather, we see the outcomes of those laws. The distinction is crucial. Outcomes are much more complicated that the laws that govern them because they do not have to respect the symmetries displayed by the laws. By this subtle interplay, it is possible to have a world which displays an unlimited number of complicated asymmetrical structures yet is governed by a few, very simple, symmetrical laws. This is one of the secrets of the universe.

Suppose we balance a ball at the apex of a cone. If we release the ball, then the law of gravitation will determine its subsequent motion. Gravity has no preference for any particular direction in the universe; it is entirely democratic in that respect. Yet, when we release the ball, it will always fall in some particular direction, either because it was given a little push in one direction, or as a result of quantum fluctuations which do not permit an unstable equilibrium state to persist. So here, in the outcome of the falling ball, the directional symmetry of the law of gravity is broken. This teaches us why science is often so difficult. When we observe the world, we see only the broken symmetries manifested as the outcomes of the laws of nature; from them, we must work backwards to unmask the hidden symmetries which characterize the laws behind the appearances.

We can now understand the answers that we obtained from the different scientists we originally polled about the simplicity of the world. The particle physicist works closest to the laws of nature themselves, and so is especially impressed by their unity, simplicity, and symmetry. But the biologist, the economist, or the meteorologist is occupied with the study of the complex outcomes of the laws, rather than with the laws themselves. As a result, it is the complexities of nature, rather than her laws, that impress them most.

Until the late 1970s, physicists focused far more upon the study of the laws, rather than the complex outcomes. This is not surprising. The study of the outcomes is a far more difficult problem that requires the existence of powerful interactive computers with good graphics for its full implementation. It is no coincidence that the study of complexity and chaos in that world of outcomes has advanced hand in hand with the growing power and availability of low-cost personal computers since the late 1970s. It has created a new methodology of experimental mathematics, dedicated to the simulation of complex phenomena, with an array of diverse applications.

(*2 709*)

Read Text C, translate it into Russian and render.

Text C

Algorithmic Information Theory

My story begins in 1686 with Gottfried W. Leibniz's philosophical essay "Discours de métaphysique" (Discourse on Metaphysics), in which he discusses how one can distinguish between facts that can be described by some law and those that are lawless, irregular facts. Leibniz's very simple and profound idea appears in section VI of the Discours, in which he essentially states that a theory has to be simpler than the data it explains, otherwise it does not explain anything. The concept of a law becomes vacuous if arbitrarily high mathematical complexity is permitted, because then one can always construct a law no matter how random and patternless the data really are. Conversely, if the only law that describes some data is an extremely complicated one, then the data are actually lawless. Today the notions of complexity and simplicity are put in precise quantitative terms by a modern branch of mathematics called algorithmic information theory. Ordinary information theory quantifies information by asking how many bits are needed to encode the information. For example, it takes one bit to encode a single yes/no answer. Algorithmic information, in contrast, is defined by asking what size computer program is necessary to generate the data. The minimum number of bits — what size string of zeros and ones — needed to store the program is called the algorithmic information content of the data. Thus, the infinite sequence of numbers 1, 2, 3, . . . has very little algorithmic information; a very short computer program can generate all those numbers. It does not matter how long the program must take to do the computation or how much memory it must use — just the length of the program in bits counts. (I gloss over the question of what programming language is used to write the program — for a rigorous definition, the language would have to be specified precisely. Different programming languages would result in somewhat different values of algorithmic information content.) To take another example, the number pi, 3.14159. . . , also has only a little algorithmic information content, because a relatively short algorithm can be programmed into a computer to compute digit after digit. In contrast, a random number with a mere million digits, say 1.341285. . . 64, has a much larger amount of algorithmic information. As the number lacks a defining pattern, the shortest program for outputting it will be about as long as the number itself.

No smaller program can calculate that sequence of digits. In other words, such digit streams are incompressible, they have no redundancy; the best that one can do is transmit them directly. They are called irreducible or algorithmically random.

How do such ideas relate to scientific laws and facts? The basic insight is a software view of science: a scientific theory is like a computer program that predicts our observations, the experimental data. Two fundamental principles inform this viewpoint. First, as William of Occam noted, given two theories that explain the data, the simpler theory is to be preferred (Occam's razor). That is, the smallest program that calculates the observations is the best theory. Second is Leibniz's insight, cast in modern terms — if a theory is the same size in bits as the data it explains, then it is worthless, because even the most random of data has a theory of that size. The simpler the theory is, the better you understand something.

(*3 422*)

Read and translate Text D into Russian.

Text D

Disorganized Complexities

Complexity, like crime, comes in organized and disorganized forms. The disorganized form goes by the name of chaos and has proven to be ubiquitous in nature. The standard folklore about chaotic systems is that they are unpredictable. They lead to out-of-control dinosaur parks and out-of-work meteorologists. However, it is important for us to appreciate the nature of chaotic systems more fully than the Hollywood headlines do.

Classical (that is, non-quantum mechanical) chaotic systems are not in any sense intrinsically random or unpredictable. They merely possess extreme sensitivity to ignorance. We are never going to get the mathematical equations for weather prediction one hundred percent correct — there is too much going on — so we will always end up being inaccurate to some extent in our predictions.

Another important feature of chaotic systems is that, although they become unpredictable when you try to determine the future from a particular uncertain starting value, there may be a particular stable statistical spread of outcomes after a long time, regardless of how you started out. The most important thing to appreciate about these stable statistical distributions of events is that they often have very stable and predictable average behaviors. As a simple example, take a gas of moving molecules (their average speed of motion determines what we call the gas's "temperature"), and think of the individual molecules as little balls. The motion of any single molecule is chaotic, because each time it bounces off another molecule any uncertainty in its direction is amplified exponentially. This is something you can check for yourself by observing the collisions of marbles or snooker balls.

Gas molecules behave like a huge number of snooker balls bouncing off each other and the denser walls of their container. One knows from bitter experience that snooker exhibits sensitive dependence on initial conditions: a slight miscue of the cue ball produces a big miss! Unlike the snooker balls, the molecules won't slow down and stop. Their typical distance between collisions is about 200 times their radius. With this value of d/r, the unpredictability grows 200-fold at each close molecular encounter. All the molecular motions are individually chaotic, just like the snooker balls, but we still have simple rules like Boyle's Law, governing the pressure P, volume V, and temperature T — the averaged properties — of a confined gas of molecules: PV/T = constant. The lesson of this simple example is that chaotic systems can have stable, predictable, long-term, average behaviors. However, it can be difficult to predict when, because the mathematical conditions that are sufficient to ensure it are often very difficult to prove. You usually just have to explore numerically to discover whether the computation of time averages converges towards a steady behavior in a nice way or not.

(*2901*)

Read Text E and give a short summary.

Text E

On the Edge of Chaos

The advent of small, inexpensive, powerful computers with good interactive graphics has enabled large, complex, and disordered situations to be studied observationally — by looking at a computer monitor. Experimental mathematics is a new tool. A computer can be programmed to simulate the evolution of complicated systems, and their long-term behavior observed, studied, modified, and replayed. By these means, the study of chaos and complexity has become a multidisciplinary subculture within science. The study of the traditional, exactly soluble problems of science has been augmented by a growing appreciation of the vast complexity expected in situations where many competing influences are at work. Prime candidates are provided by systems that evolve in their environment by natural selection, and in so doing modify those environments in complicated ways.

As intuition about the nuances of chaotic behavior has matured by exposure to natural examples, novelties have emerged that give important hints about how disorder often develops from regularity. Chaos and order have been found to coexist in a curious symbiosis. Imagine a very large egg timer in which sand is falling, grain by grain, to create a growing sand pile. The pile evolves under the force of gravity in an erratic manner. Sandfalls of all sizes occur, and their effect is to maintain the overall gradient of the sand pile in a temporary equilibrium, always just on the verge of collapse. The pile steadily steepens until it reaches a particular slope, and then gets no steeper. This self-sustaining process was dubbed "self-organising criticality". It is always about to experience an avalanche of some size or another. The sequence of events that maintains its state of large-scale order is a slow local build of sand somewhere on the slope, then a sudden avalanche, followed by another slow build up, a sudden avalanche, and so on. At first the infalling grains affect a small area of the pile, but gradually their avalanching effects increase to span the dimensions of the entire pile, as they must if they are to organize it.

At a microscopic level, the fall of sand is chaotic, yet the result in the presence of a force like gravity is large-scale organization. If there is nothing peculiar about the sand that renders avalanches of one size more probable than all others, then the frequency with which avalanches occur is proportional to some mathematical power of their size (the avalanches are said to be "scale-free" processes). There are many natural systems (like earthquakes) and man-made ones (like stock market crashes) where a concatenation of local processes combines to maintain a semblance of equilibrium in this way. Order develops on a large scale through the combination of many independent, chaotic, small-scale events that hover on the brink of instability. Complex adaptive systems thrive in the hinterland between the inflexibilities of determinism and the vagaries of chaos. There, they get the best of both worlds: out of chaos springs a wealth of alternatives for natural selection to sift through, while the rudder of determinism sets a clear average course towards islands of stability.

Originally it was believed that the way in which the sandpile organized itself might be a paradigm for the development of all types of organized complexity. This was too optimistic. But it does provide clues as to how many types of complex system organize themselves. The avalanches of sand can represent extinctions of species in an ecological balance, jams on a motorway traffic flow, the bankruptcies of businesses in an economic system, earthquakes or volcanic eruptions in a model of the pressure equilibrium of the Earth's crust, and even the formation of oxbow lakes by a meandering river. Bends in the river make the flow faster there, which erodes the bank, leading to an oxbow lake forming. After the lake forms, the river is left a little straighter. This process of gradual buildup of curvature followed by sudden oxbow formation and straightening is how a river on a flat plain "organizes" its meandering shape.

At first, it was suggested that this route to self-organization might be followed by all complex self-adaptive systems. That was far too optimistic: it is just one of many types of self-organization. Yet the nice feature of these insights is that they show that it is still possible to make important discoveries by observing the everyday things of life and asking the right questions. Sometimes complexity can be simple too.

(*4757*)

Read and translate Text F without a dictionary

Text F

The Number Omega

The first step on the road to omega came in a famous paper published precisely 250 years after Leibniz's essay. In a 1936 issue of the Proceedings of the London Mathematical Society, Alan M. Turing began the computer age by presenting a mathematical model of a simple, general-purpose, programmable digital computer. He then asked: "Can we determine whether or not a computer program will ever halt?" This is Turing's famous halting problem. Of course, by running a program you can eventually discover that it halts, if it halts. The problem, and it is an extremely fundamental one, is to decide when to give up on a program that does not halt. A great many special cases can be solved, but Turing showed that a general solution is impossible. No algorithm, no mathematical theory, can ever tell us which programs will halt and which will not.

The next step on the path to the number omega is to consider the ensemble of all possible programs. Does a program chosen at random ever halt? The probability of having that happen is my omega number. First, I must specify how to pick a program at random. A program is simply a series of bits, so flip a coin to determine the value of each bit. How many bits long should the program be? Keep flipping the coin so long as the computer is asking for another bit of input. Omega is just the probability that the machine will eventually come to a halt when supplied with a stream of random bits in this fashion. (The precise numerical value of omega depends on the choice of computer programming language, but omega's surprising properties are not affected by this choice. And once you have chosen a language, omega has a definite value, just like pi or the number 3.) Being a probability, omega has to be greater than 0 and less than 1, because some programs halt and some do not. Imagine writing omega out in binary. You would get something like 0.1110100…. These bits after the decimal point form an irreducible stream of bits. They are our irreducible mathematical facts (each fact being whether the bit is a 0 or a 1).

Omega can be defined as an infinite sum, and each N-bit program that halts contributes precisely 1/2N to the sum. In other words, each N-bit program that halts adds a 1 to the Nth bit in the binary expansion of omega. Add up all the bits for all programs that halt, and you would get the precise value of omega. This description may make it sound like you can calculate omega accurately, just as if it were the square root of 2 or the number pi. Not so — omega is perfectly well defined and it is a specific number, but it is impossible to compute in its entirety.

(*2628*)

Read and smile

Mathematics is the art of giving the same name to different things. — *J. H. Poincare*.

An engineer thinks that his equations are an approximation to reality. A physicist thinks reality is an approximation to his equations. A mathematician doesn't care.

“Mathematicians are like Frenchmen: whatever you say to them, they translate it into their own language, and forthwith it means something entirely different.” *Goethe*.

Unit 3

Texts:

A. Some Mathematical Tools for Information Processing
B. Algebraic Equations. Construction of Roots
C. The Unreasonable Effectiveness of Mathematics in Science and Engineering
D. Accurate Reconstruction of Discontinuous Functions
E. Function Approximation and Functional Optimization
F. Mathematics on the Web

Preliminary exercises

I. Read Text A and choose the relevant meaning of the following words:

face, *n* — лицо, выражение лица, внешний вид, наглость, нахальство, поверхность, внешняя сторона, грань, лицевая сторона, циферблат, грань, облицовка;

face, *v* — быть обращенным к …, смотреть в лицо, сталкиваться, облицовывать, полировать, скомандовать поворот

volume, *n* — том, книга, объем, количество, масса (вещества), емкость, вместительность, сила, интенсивность, полнота, громкость

impact, *n* — удар, толчок, столкновение, импульс;

impact, *v* — плотно сжимать уплотнять, прочно укреплять, ударять

solution, *n* — растворение, распускание, раствор, растворенное состояние, решение, объяснение, разрешение (*сомнений*), окончание (*болезни*), исполнение (обязательства)

instill, *v* — постепенно внушать, прививать (*чувство*), внушать, воспитывать, исподволь учить, вводить малыми дозами

concept, *n* — понятие, идея, общее представление, концепция

core, *n* — сердцевина, ядро, внутренность, суть, сущность, стержень, жила (*кабеля*), активная зона реактора, основная часть, основная идея approach, *n* — приближение, приход, наступление, подступы, подход, метод, подача, подвод, разбег

approach, *v* — подходить, приближаться, близиться, граничить, обращаться;

introduce, *v* — вводить, вставлять, помещать,, впускать, устанавливать, учреждать, вносить на рассмотрение, представлять, знакомить, ознакомлять, приступать, начинать, внедрять

II. Read Text A and find equivalent phrases in the right-hand column. Find them in the text:

1) аппроксимация по методу наименьших квадратов	a) notion of “frequency”
2) основная часть математического инструментария	b) multiresolution analysis
3) частотный метод	c) trigonometric series representations
4) узкое место	d) least-squares approximation
5) прогрессивная передача	e) core of the mathematical toolbox
6) понятие "частотности"	f) frequency approach
7) представление в виде тригонометрического ряда	g) bottleneck
8) многомасштабный анализ	h) progressive transmission

III. Memorize the following basic vocabulary and terminology to Text A:

exponential growth — экспоненциальный рост, степенной рост

innovative solution — перспективное решение, инновационное решение

high-dimensional space — многомерное пространство

partial differential equation — дифференциальное уравнение в частных производных

least-squares approximation — приближение по методу наименьших квадратов

data dimensionality reduction — понижение размерности данных

data compression — уплотнение данных

discrete Fourier transform — дискретное преобразование Фурье

fast Fourier transform — быстрое преобразование Фурье

wavelet — вейвлет, всплеск

multi-scale processing — многомасштабная обработка
multiresolution analysis — многомасштабный анализ
strong background — обширный опыт, хорошая подготовка
principal component analysis — метод главных компонент

Text A

Some Mathematical Tools for Information Processing

Over the past decade, we have been facing a rapidly increasing volume of "information" contents to be processed and understood. The trend of exponential growth of easily accessible information is certainly going to continue well into the twenty-first century, and the bottleneck created by this information explosion will definitely require innovative solutions from the scientific and engineering communities, particularly those technologists with better understanding of, and strong background in, applied mathematics. In this regard, the "queen of sciences" must extend her influence and impact by providing innovative theory, methods, and algorithms to virtually every discipline, far beyond sciences and engineering, for processing, transmitting, receiving, understanding, and visualizing data sets, which could be very large or live in some high-dimensional space.

Of course the basic mathematical tools, such as partial differential equations (PDE) methods and least-squares approximation introduced by Gauss, are always among the core of the mathematical toolbox for applied mathematics. But other innovations and methods must be integrated in this toolbox as well. Linear algebra is extended to "linear analysis" with applications to principal component analysis (PCA) and data dimensionality reduction (DDR). For data compression, the notion of entropy is introduced to quantify coding efficiency. One of the most essential ideas is the notion of "frequency" of the data information. A contemporary of Gauss, by the name of Joseph Fourier, instilled this important concept to our study of physical phenomena by his innovation of trigonometric series representations, along with powerful mathematical theory and methods, to significantly expand the core of the toolbox for applied mathematics. Discrete Fourier transform (DFT) followed by an efficient computational algorithm, called fast Fourier transform (FFT), as well as a real-valued version of the DFT, called discrete cosine transform (DCT) are considered, with application to extracting frequency content of the given discrete data set that facilitates reduction of the entropy and thus significant improvement of the coding efficiency. DFT can be viewed as a discrete version of the Fourier series. The integral version of the sequence of Fourier coefficients is called the Fourier transform (FT). Another important idea is the “multi-scale” structure of data sets. Less than three decades ago, with the birth of another exciting mathematical subject, called “wavelets,” the data set of information can be put in the wavelet domain for multi-scale processing as well.

Multi-scale data analysis is introduced and compared with the Fourier frequency approach, and the architecture of multiresolution analysis (MRA) is applied to the construction of wavelets and formulation of the multi-scale wavelet decomposition and reconstruction algorithms.

(*2 916*)

Task 1. ***Answer the following questions.***

1. Why are the new methods of information processing required now?
2. What is the role of mathematics in this regard?
3. What is the core of the mathematical toolbox for applied mathematics?
4. What is the concept of Joseph Fourier?
5. What new methods are applied for information processing?

Task 2. ***Translate the following sentences into English.***

1. Известно, что в двадцать первом веке объем информации, которую необходимо обработать, будет существенно возрастать. 2. Студентам необходимо изучить основы теории и методы прикладной математики, применяемые для обработки информации. 3. Современник Гаусса, Джозеф Фурье, известен своими работами по теории рядов. 4. Для количественной оценки эффективности кодирования вводится понятие энтропии. 5. Считается, что уравнения в частных производных — это один из основных математических инструментов.

Task 3. ***Read the following expressions, consulting the SUPPLEMENT.***

$2\frac{3}{7}$; 0.28; 16.27

$4a^2 - 20a = 25$; $7^{a+1} + 6^2 = 3a$; $\sqrt{4a+3} + 2a = 1$; $a = \sqrt[3]{27} + \sqrt[3]{1}$;

$\frac{a^2 - 2ab + b^2}{a - b} = a - b$;

$log\ N^m = m\ log\ N$; a $\sqrt{\frac{(b-c^2)-2b}{6}} = 18$; $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$;

$\int\sqrt{a^2-x^2}\,dx$; $\int_0^2 (x^3 - x^2)\,dx$; $\int_0^{\frac{\pi}{4}} tg^2 dx$.

Grammar Revision: *Suffix "-ing"*

Grammar task 1. *Find the words with suffix "-ing" in Text A and explain them.*

Grammar task 2. *Translate the following sentences into Russian paying attention to the words with suffix "-ing".*

1. A new technique having been worked out, the yields rose.
2. Other theories having so far proved inadequate, dynamo theories of the origin of solar fields are regarded as the most promising.
3. All rock species yet tested are somewhat radioactive, the radioactivity being accompanied by the evolution of heat.
4. They took all the measurements during actual operation of the machine,this being the usual practice in those days.
5. If the savings in operating costs is to be fully realized, a high standard of reliability is necessary.
6. Up to present time, several writers have succeeded in finding exact solutions of the fundamental differential equations in certain particular cases.
7. Life is a matter of making wise choices – of knowing when to draw the line.
8. The method as developed by W.R. Evens is indicating the location of roots of the characteristic equation.
9. Objectives may involve either getting something one does not have or giving up none or as little as possible of something one does have.
10. Never having encountered friends to drop in simultaneously, she was almost totally alone.

Read Text B and give a short summary.

Text B

Algebraic Equations. Construction of Roots

Decartes first states (without proof) that maximum number of roots that an equation "can have" is equal to its "dimension" (degree). When the total number of "true" (positive) and "false" (negative) roots is less than the dimension, one can according to Decartes, artificially add "imaginary" roots, a naïve term coined by him but not defined. He also proves that for a polynomial P to be divisible by (X-a) it is necessary and sufficient that P(a) be zero. He then uses his indeterminate coefficients to describe the division of a polynomial by (X-a). So it was important for him to know at least one root. For an equation with rational coefficients, he studies the rational roots if any.

Decartes was also interested in the number of real roots, and asserted without proof that the maximum number of positive roots of an equation is equal to the number of alternations of the sings "+" and "-" between consecutive nonzero coefficients, while the maximum number of negative roots is equal to the number of times the signs do not alternate in the same sequence. This is the celebrated "rule of signs", which earned unfounded criticism for Decartes. The result was proved in the eighteenth century, in particular by de Gua and Segner, and led to the "final" theorem of Sturm.

The utility of solving algebraic equations arose from Pappus's problem. Constructing the roots geometrically was a consequence of Decartes's constructivist conception of knowledge. By intersecting a circle and a parabola (auxiliary curves), Decartes could solve the trisection of the angle and the duplication of the cube (two ancient problems leading to cubic equations) and, from there, general equations of degree 3 and 4. For higher degrees, Decartes introduced more complicated algebraic auxiliary curves, including a specific cubic one. In modern terms, the method regarded an algebraic equation $H(x) = 0$ as the resultant of eliminating y between $F(x, y) = 0$ and $G(x, y) = 0$. To construct the solutions of $H(x) = 0$, it suffices to make a suitable choice of F and G and then to study graphically the abscissa of the points of intersection of the curves for $F = 0$ and $G = 0$, the skill of the geometer lying in the "most simple" choice of F and G.

(*2212*)

Read Text C and put your own questions to the text. Discuss the questions with the group.

Text C

The Unreasonable Effectiveness of Mathematics in Science and Engineering

The remarkable efficiency of mathematics in predicting the behavior of physical systems has fascinated many scientists.

Einstein is said to have remarked that "The most incomprehensible thing about the universe is that it is comprehensible". This observation was elaborated by Eugene Wigner in his famous paper in Pure Mathematics entitled "The Unreasonable Effectiveness of Mathematics in the Natural Sciences".

The theme was further developed in "The Unreasonable Effectiveness of Mathematics" by R. W. Hamming, which considered the predictive, as well as descriptive powers, of mathematics in relation to engineering.

Two surprising conclusions appear from these papers:

(i) Although it is a product of the human mind, mathematics is also involved in some strange metaphysical way at the deepest levels of physical existence. To quote Wigner:

"The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning".

(ii) There is no Darwinian explanation for the presence of mathematical abilities within the mind. The ability to understand physics could not have arisen by evolution. Although our bodies may well be the product of random mutation and selection all the way from amoeba to man, our minds have some "unevolved" dimension. To quote Hamming:

"But it is hard for me to see how simple Darwinian survival of the fittest would select for the ability to do the long chains that mathematics and science seem to require".

"If you pick 4,000 years for the age of science, generally, then you get an upper bound of 200 generations. Considering the effects of evolution we are looking for via selection of small chance variations, it does not seem to me that evolution can explain more than a small part of the unreasonable effectiveness of mathematics”.

Or Wigner again: “Certainly it is hard to believe that our reasoning power was brought, by Darwin's process of natural selection, to the perfection which it seems to possess”.

So we are left with something of a mystery. According to the materialist worldview, the mind (including mathematicians' minds) is an epiphenomenon of matter which has evolved solely to ensure the survival of the selfish genes which code for it. So why should this “top-level” phenomenon have such intimate access to the “bottom level” phenomena such as quantum physics? After all, the two levels are supposedly separated by less well-understood (in some cases) explanatory layers such as evolutionary psychology, neurology, cell biology, genetics, molecular biology, and chemistry.

(*2829*)

Read Text D, translate it into Russian and render.

Text D

Accurate Reconstruction of Discontinuous Functions

Approximation of smooth functions by Fourier series or by truncated orthogonal polynomial expansions in general is known to be exponentially convergent and highly accurate. For functions with singularities, however, convergence of a partial sum of orthogonal series is adversely affected in the area over which the singularities occur, a problem which has come to be known as the Gibbs phenomenon. This phenomenon manifests in an oscillatory behavior at the vicinity of the jumps and thus presents an obstruction in the reconstruction of a discontinuous function. An exposition on the nature of the Gibbs phenomenon and some remediation schemes to counter its effect can be found. A class of techniques aimed at resolving the Gibbs phenomenon comprises Padé-type approximations. These methods extend the standard Padé approximation by making use of orthogonal polynomials as basis in lieu of the canonical basis with which the numerator and denominator of a Padé approximant are expanded. A Padé-type approximant enjoys the advantage of utilizing rational functions, which are broader than polynomials and can have singularities, and hence there is a stronger likelihood that it will capture the singularities of the function being approximated. Some Padé-based methods work without requiring information about the jump locations. However, locating jump discontinuities can become a relevant issue when the actual function is not explicitly known. In many cases, for instance, involving spectral approximations of non smooth solutions to some partial differential equations, the solution comes in the form of computational data that are contaminated by Gibbs phenomenon. As these data are noisy, the standard procedure is to postprocess them to correct the phenomenon. One way this can be done is to use Padé-type approximation. This Padé postprocessing approach, however, may turn out to be less successful unless fed with some information about the possible jump positions which can be advantageous for its effective implementation. As computational data may not show explicitly the existence and whereabouts of possible jumps, to somehow locate them can become imperative. A study by Driscoll and Fornberg reveals just how significant the knowledge of the jump locations can be in correcting the Gibbs phenomenon. Realizing that the poles available in a rational approximant do not intrinsically and adequately reproduce the jump behaviors of a discontinuous functions, they devised an approach that incorporates the jump locations into the approximation process.

(*2567*)

Read and translate Text E into Russian.

Text E

Function Approximation and Functional Optimization

In functional optimization problems, also known as infinite programming problems, functionals have to be minimized with respect to functions belonging to subsets of function spaces. Function-approximation problems, the classical problems of the calculus of variations and, more generally, all optimization tasks in which one has to find a function that is optimal in a sense specified by a cost functional belong to this family of problems. Such functions may express, for example, the routing strategies in communication networks, the decision functions in optimal control problems and economic ones. Experience has shown that optimization of functionals over admissible sets of functions made up of linear combinations of relatively few basis functions with a simple structure and depending nonlinearly on a set of “inner” parameters (e.g., feed forward neural networks with one hidden layer and linear output activation units) often provides surprisingly good suboptimal solutions. In such approximation schemes, each function depends on both external parameters (the coefficients of the linear combination) and inner parameters (the ones inside the basis functions). These are examples of variable-basis approximators since the basis functions are not fixed but their choice depends on the one of the inner parameters. In contrast, classical approximation schemes (such as the Ritz method in the calculus of variations) do not use inner parameters but employ fixed basis functions, and the corresponding approximators exhibit only a linear dependence on the external parameters. Then, they are called fixed-basis or linear approximators. Certain variable-basis approximators can be applied to obtain approximate solutions to functional optimization problems. This technique was later formalized as the extended Ritz method (ERIM) and was motivated by the innovative and successful application of feed forward neural networks in the late 80 s. The basic motivation to search for suboptimal solutions of these forms is quite intuitive: when the number of basis functions becomes sufficiently large, the convergence of the sequence of suboptimal solutions to an optimal one may be ensured by suitable properties of the set of basis functions, the admissible set of functions, and the functional to be optimized. Computational feasibility requirements (i.e., memory occupancy and time needed to find sufficiently good values for the parameters) make it crucial to estimate the minimum number of computational units needed by an approximator to guarantee that suboptimal solutions are “sufficiently close” to an optimal one. Such a number plays the role of “model complexity” of the approximator and can be studied with tools from linear and nonlinear approximation theory. As compared to fixed-basis approximators, in variable-basis ones the nonlinearity of the parametrization of the variable basis functions may cause the loss of useful properties of best approximation operators, such as uniqueness, homogeneity, and continuity, but may allow improved rates of approximation or approximate optimization. Then, to justify the use of variable-basis schemes instead of fixed-basis ones, it is crucial to investigate families of function-approximation and functional optimization problems for which, for a given desired accuracy, variable-basis schemes require a smaller number of computational units than fixed-basis ones.

(*3423*)

Read and translate Text F without a dictionary.

Text F

Mathematics on the Web

Over the past several years, a project has been quietly evolving that has important implications for those interested in using mathematical notation within webpages in a way that not only displays that mathematics beautifully but allows it to interact with other applications and environments. That project is MathJax, and it is an attempt to provide a universal, industrial-strength, math-on-the-web solution that is standards-based and applicable. Current users include publishers of large-scale scientific websites, bloggers and social networkers, users of course-management systems, and individual faculty members who just want to post their homework assignments easily online.

MathJax is an open-source project, drawing on the talents of a variety of individuals. Anyone who has tried to include mathematical notation in a webpage knows that this is not an easy task. The traditional solution is to use images of the equations and link those into the page to represent the mathematics. This is a cumbersome approach that has a number of drawbacks (it is hard to get the images to match the surrounding text, they don't scale or print well, they cannot be easily copied, and so on). The Mathematical Markup Language (MathML) was intended to solve this problem, but for a variety of reasons, more than a decade after its specification was released, most of the major browsers still don't support it. The MathJax project plugs the gap left by a lack of browser support for MathML, making it possible for mathematicians — and the scientific community at large — finally to take advantage of the MathML standard and all it implies.

MathJax is being developed as a platform for mathematics in webpages that works across all the major browsers (including mobile devices such as the iPad, iPhone, and Android phones). It allows authors to write their equations using several formats, including MathML and TEX, and displays the results using MathML in those browsers that support it.

MathJax does not require the viewer to download any software (though it will take advantage of certain locally installed fonts when they are present), and since it uses actual fonts, its output scales and prints better than math presented as images. Because the pages include the original TEX or MathML markup, search engines can index the mathematics within them. Since there are no images to create, the mathematics on the page can be dynamically generated and can include links and other interactive content.

Read and smile

I do not think — therefore I am not.

Old mathematicians never die; they just lose some of their functions.

Classification of mathematical problems as linear and nonlinear is like classification of the Universe as bananas and non-bananas.

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SUPPLEMENT

Mathematical Symbols and Operations

Σ	— summation
dx	— differential of x
$\frac{dy}{dx}$	— derivative of y with respect to x
$\frac{\partial y}{\partial x}$	— partial derivative of y with respect to x
$f(x)$	— function of x
lim	— limit
$\lim_{x \to 5} f(x)$	— limit $f(x)$ as x tends to 5
$log_5 a$	— logarithm of a to the base 5
lg	— decimal logarithm
ln	— logarithm natural
$\int$	— integral of
$\int f(x)dx$	— integral of a function of x over dx
$\int_n^m f(x)dx$	— integral of a function of x over dx between limits n and m
sin	— sine
cos	— cosine
tan, tg	— tangent
cjt, ctg	— cotangent

ADDITION — СЛОЖЕНИЕ

Add —	Прибавить, складывать
Added —	Слагаемое
Item —	Слагаемое
Sum —	Сумма, суммировать
Summand —	Слагаемое

Total —	Сумма, итог, целый, подводить итог
Quantity —	Количество, величина
Unknown —	Неизвестное
Equality —	Равенство

Example: $a + b = c$

Читается, как: $a + b$ equals c; $a + b$ is equal to c; $a + b$ makes c; $a + b$ is c.

SUBTRACTION — ВЫЧИТАНИЕ

Subtract —	Вычитать
Less —	Без, минус, за вычетом
Minuend —	Уменьшаемое
Subtracted —	Вычитаемое
Difference —	Разность
Negative —	Отрицательный

Example: $a - b = c$

Читается, как: $a - b$ equals c; $a - b$ is equal to c; b from a leaves c; a diminished by b is c.

MULTIPLICATION — УМНОЖЕНИЕ

Multiply —	Умножить
Multiplicand —	Множимое
Multiplier —	Множитель
Factor —	Множитель, коэффициент
Product —	Произведение

Examples:

$1 \times 1 = 1$ Читается, как: once one is one

$2 \times 2 = 4$ Читается, как: twice two is four

$3 \times 3 = 9$ Читается, как: three times three is nine

$a x b = c$ Читается, как: a (multiplied) by b equals c.

DIVISION — ДЕЛЕНИЕ

Divide —	Делить
Divided —	Делимое

Divisor —	Делитель
Quotient —	Частное, отношение
Reminder —	Остаток

Examples:

$a : b = c$ Читается, как: *a* divided by *b* is equal to *c*.

$\frac{a+b}{a-b} = \frac{c+d}{c-d}$ Читается, как: *a* plus *b* over *a* minus *b* is equal to *c* plus *d* over *c* minus *d*.

FRACTIONS — ДРОБИ

Common fractions — простые дроби

Numerator —	Числитель
Denominator —	Знаменатель
Integer —	Целое число
Cardinal number —	Количественное числительное
Ordinal number —	Порядковое числительное
Nought —	Ноль (в математических выражениях)
Zero —	Ноль (на шкалах)

Examples:

$\frac{1}{2}$ Читается, как: one half, a half

$\frac{2}{4}$ Читается, как: two fourth

$5\frac{2}{7}$ Читается, как: five and two seventh

Decimal fractions

В Англии и Америке знаки десятичных дробей отделяют точкой — point. Каждая цифра читается отдельно. Ноль читается любым из трех способов: Ноль целых можно совсем не читать, а только читать “point”.

Examples:

0.2 Читается, как: O point two; point two; zero point two; nought point two.

34.86 Читается, как: thirty four point eight six.

INVOLUTION — ВОЗВЕДЕНИЕ В СТЕПЕНЬ

Power —	Степень, показатель степени
Base —	Основание
Raise to the power —	Возводить в степень
Exponent —	Показатель
Square —	Квадрат, возводить в квадрат
Cube —	Куб, возводить в куб
Even —	Четный
Even form —	Четная степень
Odd —	Нечетный
Odd form —	Нечетная степень

Examples:

5^2 Читается, как: five squared; five square; five raised to the second power; five to the power two; the second power of five.

x^{-5} Читается, как: *x* to the minus fifth (power)

y^7 Читается, как: *y* to the seventh (power)

EVOLUTION — ИЗВЛЕЧЕНИЕ КОРНЯ

Root —	Корень
Extract the root of (out of) —	Извлечь корень из
Index —	Показатель
Index laws —	Правила действий с показателями
Indices —	Показатели
Radical sign —	Знак корня

Examples:

$\sqrt{9} = 3$ Читается, как: the square root of nine is three.

$\sqrt[5]{a^7}$ Читается, как: the fifth root out of *a* to the power seven.

PROPORTION — ПРОПОРЦИЯ

Term —	Член
Expression —	Выражение
Extremes —	Крайние члены пропорции
Means —	Средние члены пропорции
Mean —	Средний, среднее значение
Proportional —	Пропорциональный, член пропорции

Directly — Прямо, непосредственно
Inversely — Обратно
Vary — Меняться
Vary directly (inversely) — Меняться прямо (обратно) пропорционально
Constant — Постоянная величина, константа

Examples:

$a : b = c : d$ Читается, как: *a* is to *b* as *c* is to *d.*

$x = ky$ Читается, как: *x* varies directly to *y; x* is directly proportional to *y.*

$x = \frac{k}{y}$ Читается, как: *x* varies inversely to *y; x* is inversely proportional to *y.*

EQUATION — УРАВНЕНИЕ

Formula — Формула
Formulae, formulas — Формулы
Algebraic(al) — Алгебраический
Value — Величина, значение
Identity — Тождество

Examples:

$(a + b)(a - b) = a^2 - b^2$ Читается, как: the product of the sum and difference of a and b is equal to the difference of their squares.

EXAMPLES OF READING FORMULAS

$2 + x + \sqrt{4} + x^2 = 10$ Читается, как: two plus *x* plus the square root (out) of four plus *x* squared is equal ten

$v = u\sqrt{sin^2 i - cos^2 i} = u$ Читается, как: *v* is equal to u square root out of sine square *i* minus cosine square *i* is equal to *u*

$a^{m/n} = \sqrt[n]{a^m}$ Читается, как: *a* to the *m/n* –th power is equal to the *n*-th root of the *a* to *m*-th power

$\int \frac{dx}{\sqrt{a^2 - x^2}}$ Читается, как: integral of dx over (divided by) the square root out of a square minus x square

$\int_a^b f(x)\,dx$ Читается, как: the integral of f of x over dx from a to b.

$\int_0^1 \sqrt{x}\,dx = \int_0^1 x^{\frac{1}{2}}\,dx = F(1) - F(0) = \frac{2}{3}$ Читается, как: the integral of square root of x over dx on the interval [0, 1] is equal to the integral of x to the power one second over dx is equal to function of 1 minus function of zero is equal to two thirds.

$\log \frac{N_1}{N_2} = \log N_1 - \log N_2$ Читается, как: logarithm of N first over N second is equal to logarithm of N first minus logarithm of N second.

CONTENTS

Учебное пособие

Дикова Ольга Дмитриевна
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Обучение чтению литературы на английском языке по специальности «Прикладная математика»

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Компьютерная верстка *Г.Ю. Молотковой*

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